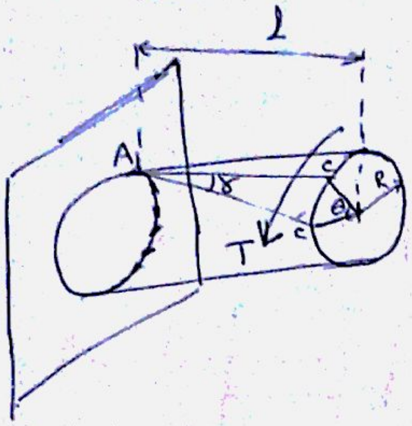
Lec:7 د/عامر Machine Design (1) 7-4-2015

Torsion of strength bars of circular sections



$$\gamma = \frac{CC'}{AC} = \frac{R\theta}{L}$$

$$\tau = G\gamma = G\frac{R\theta}{L}$$

* $\boxed{T = FA}$
* $\boxed{T = \tau A r}$

$$\tau = \frac{G\theta r}{L}$$

$$T = \int_A \tau \, dA \, r = \frac{G\theta}{l} r\,dA r$$

$$= \frac{G\theta}{L} \int r^2 dA \rightarrow J$$

$$T = \frac{G\theta J}{l}$$

$$\boxed{\frac{T}{J} = \frac{G\theta}{L}}$$

$$\boxed{\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{l}}$$

$$\tau = \frac{Tr}{J}$$

$$\tau_{max} = \frac{TR}{J} \quad R = r_{max} \quad \Rightarrow \boxed{\tau_{max} = \frac{16T}{\pi D^3}}$$

- The max stress is proportional to the applied Torque, T & inversed proportional to cube of diameter of the shaft

$$\theta = \frac{TL}{JG}$$

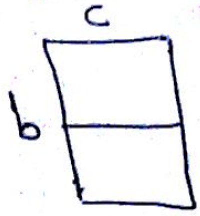
Hollow shaft

$$J = \frac{\pi}{32}(D_o^4 - D_i^4) \quad \text{and} \quad r_{max} = \frac{D_o}{2}$$

$$\tau_{max} = \frac{T}{\frac{\pi}{32}(D_o^4 - D_i^4)}\left(\frac{D_o}{2}\right) = \frac{16\,T D_o}{\pi(D_o^4 - D_i^4)}$$

$$= \frac{16\,T}{\pi D_o^2 (1-K^4)} \quad \text{where } K = \frac{D_i}{D_o}$$

Torsion of rectangular sections



$$\tau = \frac{T}{\alpha b c^2}$$

$$\theta = \frac{T}{\beta G b c^3} \qquad \int \alpha, \beta, \frac{b}{c}$$

$$\tau_{max} = \frac{T}{bc^2}\left(3 + 1{,}8\,\frac{c}{b}\right)$$

For thin sheet $\frac{b}{c}$ very large $\quad \alpha$ and $\beta = \boxed{0{,}33}$

$$\tau = \frac{3T}{bc^2}$$

$$\theta = \frac{3T}{Gbc^3}$$

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٢ - ٢١

Thin cylinders & spheres
subjected to internal pressure

$D \ggg t$

a) thin cylinder

- hoop stress
- longitudial stress

(i) <u>hopp stress</u>

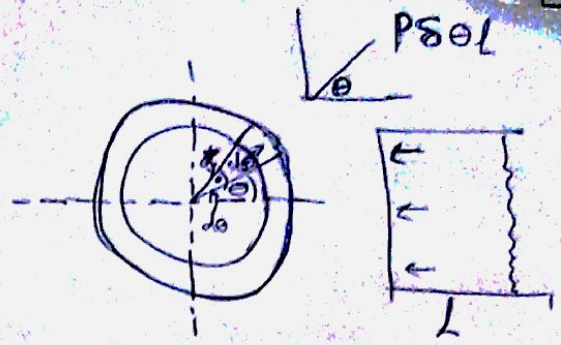
$\delta F = P(r\delta\theta)l$

$\delta F_N = Prl\,\delta\theta \sin\theta$

Normal

$= \int_0^{\pi} Prl\,d\theta . \sin\theta = 2Prl$

$= (2Lt) \times (\sigma_t)$ → material



$2Prl = 2lt\,\sigma_t$

$\sigma_t = \frac{Pr}{t} = \frac{PD}{2t}$

(ii) Long. stress

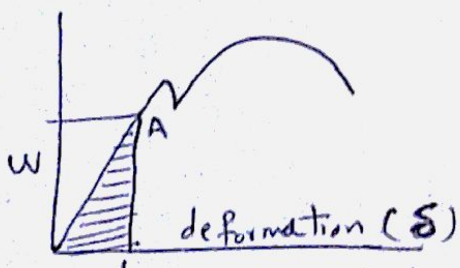
$F_a = \pi r^2 P$

$(F)_L$ material $= (2\pi r t) \times (\sigma_L)$ → Area

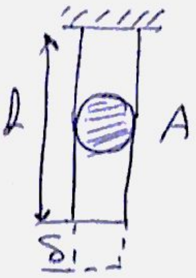
$\sigma_L = \frac{Pr}{2t} = \frac{PD}{4t}$

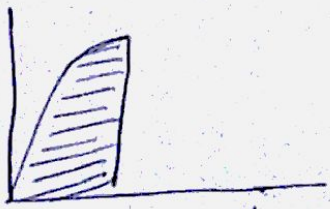
$A = 2\pi r t$

Strain Energy in torsion & comparation



low carbon steel





High carbon steel

the work done by the load is stored in the bar is called the strain energy

$V = \frac{1}{2} W \delta$

$$E = \frac{\text{Stress}}{\text{strain}}$$

$$\text{strain} = \frac{\text{stress}}{E} \;,\; \delta = \frac{1}{E}\,\frac{W\ell}{A}$$

$$\delta = \frac{W\ell}{AE}$$

$$V = \frac{W^2 L}{2AE}$$

$$\sigma = \frac{W}{A} \quad , \quad V = \frac{\sigma^2}{2E} A\ell$$

Impact stresses $\qquad\qquad$ $W \times \delta$

The work done by external load will be $(W \cancel{\times} \delta)$

$$W\delta = \frac{1}{2}\,\frac{\sigma^2}{E} AL$$

$$W(\epsilon L) = \frac{1}{2}\,\frac{\sigma^2}{E} AL$$

$$W\left(\frac{\sigma}{E}\right) = \frac{1}{2}\,\frac{\sigma^2}{E} A$$

$$\boxed{\sigma = \frac{2W}{\cancel{2}A}}$$
